Correction – Évaluation Calcul d'intégrales

Sujet A

1.

$$\int_0^2 (x^2 - 3x + 4)\,dx = \left[\frac{1}{3}x^3 - \frac{3}{2}x^2 + 4x\right]_0^2$$
$$= \frac{8}{3} - 6 + 8 = \frac{8}{3} + 2 = \frac{14}{3}$$

2. on utilise : pour tout x, $(\ln(x^2+4))' = \frac{2x}{x^2+4}$.

$$\int_1^3 \frac{x}{x^2+4}\,dx = \frac{1}{2}\int_1^3 \frac{2x}{x^2+4}\,dx$$

$$= \frac{1}{2}\left[\ln(x^2+4)\right]_1^3 = \frac{1}{2}\left(\ln(13) - \ln(5)\right) = \frac{1}{2}\ln(\frac{13}{5})$$

3.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\,dx = [\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = 1 - \frac{\sqrt{2}}{2}$$

4.

$$\int_0^1 e^{2x}\,dx = \left[\frac{1}{2}e^{2x}\right]_0^1 = \frac{1}{2}(e^2 - 1)$$

5.

$$\int_1^2 \left(\frac{1}{x^3} + \frac{2}{x}\right)\,dx = \left[\frac{-1}{2x^2} + 2\ln x\right]_1^2$$
$$= \left(-\frac{1}{8} + 2\ln 2\right) - \left(-\frac{1}{2}\right) = \frac{3}{8} + 2\ln 2$$

6.

$$\int_0^1 (1-2x)e^{2x}\,dx$$

IPP : $u = 1-2x$, $u' = -2$, $v' = e^{2x}$, $v = \frac{1}{2}e^{2x}$

$$= \left[(1-2x)\frac{1}{2}e^{2x}\right]_0^1 + \int_0^1 e^{2x}\,dx$$

$$= \left[\frac{1-2x}{2}e^{2x}\right]_0^1 + \left[\frac{e^{2x}}{2}\right]_0^1$$

$$= \left(\frac{-1}{2}e^2 - \frac{1}{2}\right) + \left(\frac{e^2}{2} - \frac{1}{2}\right) = -1$$

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Sujet B

1.

$$\int_0^2 (2x^2 - 3x + 4)\,dx = \left[\frac{2}{3}x^3 - \frac{3}{2}x^2 + 4x\right]_0^2 = \frac{16}{3} - 6 + 8 = \frac{22}{3}$$

2. on utilise : pour tout x, $(\ln(x^2+1))' = \frac{2x}{x^2+1}$.

$$\int_1^3 \frac{3x}{x^2+1}\,dx = \frac{3}{2}\int_1^3 \frac{2x}{x^2+1}\,dx$$

$$= \frac{3}{2}\left[\ln(x^2+1)\right]_1^3 = \frac{3}{2}\left(\ln(10) - \ln(2)\right) = \frac{3}{2}\ln\left(\frac{10}{2}\right) = \frac{3}{2}\ln(5)$$

3.

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin x\,dx = \left[-\cos x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = -\frac{1}{2} + \frac{\sqrt{3}}{2}$$

4.

$$\int_0^1 e^{3x}\,dx = \left[\frac{1}{3}e^{3x}\right]_0^1 = \frac{1}{3}(e^3 - 1)$$

5.

$$\int_1^2 \left(\frac{1}{x^4} + \frac{3}{x}\right)\,dx$$

$$= \left[\frac{-1}{3x^3} + 3\ln x\right]_1^2 = \left(-\frac{1}{24} + 3\ln 2\right) - \left(-\frac{1}{3}\right) = \frac{7}{24} + 3\ln 2$$

6.

$$\int_0^1 (2x+1)e^{-x}\,dx$$

IPP : $u = 2x+1$, $u' = 2$, $v' = e^{-x}$, $v = -e^{-x}$

$$= \left[-(2x+1)e^{-x}\right]_0^1 + \int_0^1 2e^{-x}\,dx$$

$$= \left(-3e^{-1} + 1\right) + \left[-2e^{-x}\right]_0^1$$

$$= \left(-3e^{-1} + 1\right) + (-2e^{-1} + 2) = 3 - \frac{5}{e}$$